

2023-2024-2 《复变函数与积分变换》期末参考答案

一. 选择题(每题 3 分)

1. A 2. D 3. C 4. B 5. D

二. 填空题(每题 3 分)

1) . 1; 2) . $u(x,y), v(x,y)$ 在 z_0 可微, 且满足 $u_x = v_y, u_y = -v_x$;

3) . $18\pi i$; 4) . $\frac{1}{\sqrt[4]{2}}$;

5) . $2\pi\delta(\omega); 2\pi i\delta'(\omega); 1 + 3/s$; 6) . 6;

三. 计算题(每题 6 分)

1) 解: $3^{3-i} = e^{(3-i)\ln 3} = e^{(3\ln 3 + 2k\pi) + i(6k\pi - \ln 3)}$ (3 分)

$$= e^{3\ln 3 + 2k\pi} (\cos(\ln 3) - i\sin(\ln 3)) \quad (3 \text{ 分})$$

2) 解: $f(z) = x\sqrt{x^2 + y^2} + iy\sqrt{x^2 + y^2} = u + iv$

$$\begin{aligned} u_x &= \frac{2x^2 + y^2}{\sqrt{x^2 + y^2}}, & u_y &= \frac{xy}{\sqrt{x^2 + y^2}}; \\ v_x &= \frac{xy}{\sqrt{x^2 + y^2}}, & v_y &= \frac{x^2 + 2y^2}{\sqrt{x^2 + y^2}}; \end{aligned} \quad (3 \text{ 分})$$

$u(x,y), v(x,y)$ 除原点外处处可微;

令 $u_x = v_y, u_y = -v_x$ 无解;

故函数处处不可导, 处处不解析. (3 分)

3) 解: $\ln(-2 - 5i) = \ln\sqrt{29} + i(\arctan\frac{5}{2} - \pi + 2k\pi), k \in \mathbb{Z}$ (3 分)

$$\sin 3i = \frac{e^{i \cdot 3i} - e^{-3i \cdot i}}{2i} = \frac{e^3 - e^{-3}}{2} i \quad (3 \text{ 分})$$

4) 解:

$$\begin{aligned} f(t) &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{\sin w}{w} e^{iwt} dw = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{\sin w}{w} \cos wtdw \\ &= \frac{1}{2\pi} \int_0^{+\infty} \frac{\sin(1+t)w + \sin(1-t)w}{w} dw \end{aligned} \quad (3 \text{ 分})$$

$$f(t) = \begin{cases} 0, t > 1 \\ 1/4, t = 1 \\ 1/2, -1 < t < 1 \\ 1/4, t = -1 \\ 0, t < -1 \end{cases} \quad (3 \text{ 分})$$

5) 解: 原式 = $2\pi i \operatorname{Res}(f(z), -1) = 2\pi i$. (6 分)

6) 解: $F(u(t)\cos\omega_0 t) = \frac{i\omega}{\omega_0^2 - \omega^2} + \frac{\pi}{2}(\delta(\omega - \omega_0) + \delta(\omega + \omega_0))$ (6 分)

四. 综合题

1) (9 分) 解: $L(t \sin 2t) = -\left(\frac{2}{s^2 + 4}\right)' = \frac{4s}{(s^2 + 4)^2}$ (4 分)

$$L^{-1}\left(\frac{s+3}{(s+1)(s+2)}\right) = L^{-1}\left(\frac{2}{s+1}\right) - L^{-1}\left(\frac{1}{s+2}\right) = 2e^{-t} - e^{-2t} \quad (5 \text{ 分})$$

2) (9 分) 解: 当 $2 < |z+3| < \infty$

$$f(z) = (z+3)^{-1} \frac{1}{z+3-2} = (z+3)^{-2} \frac{1}{1 - \frac{2}{z+3}} = \sum_{n=0}^{\infty} 2^n (z+3)^{-(n+2)} \quad (4 \text{ 分})$$

当 $0 < |z-1| < 2$

$$\begin{aligned} f(z) &= \frac{1}{2} \left(\frac{1}{z+1} - \frac{1}{z+3} \right) = \frac{1}{2} \left(\frac{1}{2+z-1} - \frac{1}{4+z-1} \right) \\ &= \frac{1}{2} \left(\frac{1}{2} \frac{1}{1 + \frac{z-1}{2}} - \frac{1}{4} \frac{1}{1 + \frac{z-1}{4}} \right) = \frac{1}{4} \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n} (z-1)^n - \frac{1}{8} \sum_{n=0}^{\infty} \frac{(-1)^n}{4^n} (z-1)^n \end{aligned} \quad (5 \text{ 分})$$

3) (13 分) 设 $L(y(t)) = F(s)$ 对方程取 L-变换得

$$s^2 F(s) - 1 + 3sF(s) + F(s) = 3 \frac{s}{s^2 + 1} \quad (6 \text{ 分})$$

$$F(s) = \frac{1}{s^2 + 1} \quad (2 \text{ 分})$$

$$y(t) = L^{-1}(F(s)) = \sin t \quad (5 \text{ 分})$$