

2019~2022级转专业高数试卷

(对19, 20, 21, 22级的转专业高数试题中上册真题整理得到)
编者注:2023级前转专业考试在大一学期末进行,因而转专业高数考试范围为上下册全书,而自2023级开始,转专业考试提前到4月份,范围也缩小到上册书,因而大家只需参考这部分试题即可

一、填空题 (每题 3 分)

1. 极限 $\lim_{x \rightarrow 0} \frac{2}{x} \sin x + x \sin \frac{3}{x} = \underline{\hspace{2cm}}$.
2. 曲线 $y = e^{-x^2}$ 的凸区间为 $\underline{\hspace{2cm}}$.
3. 已知函数 $f(x)$ 连续, 且 $f(x) = \frac{1}{\sqrt{1+x^2}} + \int_0^1 x f(x) dx$, 则 $\int_0^1 x f(x) dx = \underline{\hspace{2cm}}$.
4. 极限 $\lim_{x \rightarrow +\infty} \frac{\ln(x^6+7)}{\ln(x^2+4)} = \underline{\hspace{2cm}}$.
5. 设函数 $y = f(x)$ 由方程 $y = 1 - \ln(x+y) + e^y$ 确定, 则 $\frac{dy}{dx} = \underline{\hspace{2cm}}$.
6. 极限 $\lim_{n \rightarrow \infty} \frac{1}{n} \left(\sin \frac{1}{n} + \sin \frac{2}{n} + \cdots + \sin \frac{n}{n} \right) = \underline{\hspace{2cm}}$.
7. 曲线 $\begin{cases} x = 1 + t^2 \\ y = t^3 \end{cases}$ 在 $t = 2$ 的切线方程为 $\underline{\hspace{2cm}}$.
8. 函数 $y = x \cos^2 x$ 的 5 阶导数 $y^{(5)} = \underline{\hspace{2cm}}$.
9. 曲线 $y = \frac{x^3}{1+x^2} + \arctan(1+x^2)$ 的斜渐近线方程为 $\underline{\hspace{2cm}}$.

二、选择题 (每题 3 分)

1. 下列函数中, 在 $x = 0$ 处不可导的是 ().
(A) $f(x) = |x| \sin |x|$;
(B) $f(x) = |x| \sin \sqrt{|x|}$;
(C) $f(x) = \cos |x|$;
(D) $f(x) = \cos \sqrt{|x|}$.
2. 设 $f(x) = \frac{1-2e^{\frac{1}{x}}}{1+e^{\frac{1}{x}}} \arctan \frac{1}{2x}$ 则 $x = 0$ 是 $f(x)$ 的 ().
(A) 可去间断点;
(B) 无穷间断点;
(C) 跳跃间断点;
(D) 震荡间断点.

三、计算题 (每题 6 分)

1. 求极限 $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{\frac{1}{\sin^2 x}}$.

2. 设函数 $y = y(x)$ 由参数方程 $\begin{cases} x = \ln \sin t \\ y = \cos t + t \sin t \end{cases}$ 确定, 求 $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ 及 $\frac{d^2y}{dx^2} \Big|_{t=\frac{\pi}{4}}$.

3. 计算定积分 $\int_0^{\frac{\pi^2}{4}} (\sin \sqrt{x})^2 dx$.

4. 求极限 $\lim_{x \rightarrow 0} \left(\frac{\arcsin x}{x} \right)^{\frac{1}{\tan x^2}}$.

5. 计算不定积分 $\int \frac{xe^x}{\sqrt{e^x+1}} dx$.

6. 设函数 $y = y(x)$ 是由参数方程 $\begin{cases} x = at \cos t \\ y = at \sin t \end{cases}$ 所确定, 求 $\frac{d^2y}{dx^2}$.

7.求极限 $\lim_{x \rightarrow 0^+} (\cos \sqrt{x})^{\frac{\pi}{x}}$

8.计算不定积分 $\int \frac{x^3}{\sqrt{9-x^2}} dx$ 。

四、(7分)设 $F(x)$ 是 $f(x)$ 的一个原函数, 且 $F(0) = 1, F(x)f(x) = \cos 2x$, 求 $\int_0^{\frac{\pi}{2}} |f(x)| dx$ 。

五、(7分) 设平面区域 D 由曲线 $y = e^x$ 及 $y = e^x$ 在点 $(1, e)$ 处的切线和 y 轴围成, 求 (1) 区域 D 的面积; (2) 平面图形 D 绕 y 轴旋转一周所得立体的体积。

六、(6分) 设函数 $f(x)$ 在 $[a, b]$ 上可导且 $f'(a) < m < f'(b)$, 证明: 在 (a, b) 内至少存在一点 ξ 使得 $f'(\xi) = m$ 。

七、(7分) 已知 $\int_0^{+\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$, 计算积分 $\int_0^{\infty} \frac{\sin^2 x}{x^2} dx$

八、(6分) 设函数 $f(x)$ 在 $[0, 1]$ 上连续, $(0, 1)$ 内可导, 且

$$f(0) = 0, \quad f(1) = 1,$$

证明: (1) 在 $(0, 1)$ 内至少存在一点 x_0 , 使得 $f(x_0) = 2022 - 2023x_0$;
(2) 在 $(0, 1)$ 内存在不同的 ξ, η , 使得 $[1 + f'(\xi)] \cdot [2021 + f'(\eta)] = 4044$ 。

2019 ~ 2023

一、填空题

$$1. \lim_{x \rightarrow 0} \frac{2}{x} \sin x + x \sin \frac{2}{x} = \lim_{x \rightarrow 0} 2 \frac{\sin x}{x} + \lim_{x \rightarrow 0} x \sin \frac{2}{x} = 2 + 0 = 2$$

有界量.

$$2. y(x) = e^{-x^2}$$

$$y' = -2x e^{-x^2}$$

$$y'' = -2e^{-x^2} + 4x^2 e^{-x^2}$$

$$\text{令 } y'' = 0 \text{ 即 } (4x^2 - 2)e^{-x^2} = 0 \text{ 得 } x = \pm \frac{\sqrt{2}}{2}$$

$$\frac{1}{2}x \in (-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}) \text{ 时, } y''(x) < 0 \text{ 曲线凸}$$

$$\frac{1}{2}x \in (-\infty, -\frac{\sqrt{2}}{2}) \text{ 和 } x \in (\frac{\sqrt{2}}{2}, +\infty) \text{ 时, } y''(x) > 0, \text{ 曲线凹.}$$

$$\therefore \text{拐点为 } (-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$$

$$3. \text{令 } A = \int_0^1 x f(x) dx, \text{ 则 } f(x) = \frac{1}{\sqrt{1+x^2}} + A, \text{ 则 } x f(x) = \frac{x}{\sqrt{1+x^2}} + Ax$$

$$\text{两边同时积分得 } \int_0^1 x f(x) dx = \int_0^1 (\frac{x}{\sqrt{1+x^2}} + Ax) dx$$

$$\Rightarrow A = \sqrt{1+x^2} \Big|_0^1 + \frac{1}{2} A x^2 \Big|_0^1 = \sqrt{2} - 1 + \frac{1}{2} A \Rightarrow \frac{1}{2} A = \sqrt{2} - 1 \Rightarrow A = 2\sqrt{2} - 2$$

$$\text{即 } \int_0^1 x f(x) dx = 2\sqrt{2} - 2$$

$$4. \lim_{x \rightarrow \infty} \frac{\ln(x^2+7)}{\ln(x^2+4)} \stackrel{\frac{\infty}{\infty}}{=} \lim_{x \rightarrow \infty} \frac{\frac{6x^5}{x^2+7}}{\frac{2x}{x^2+4}} = \lim_{x \rightarrow \infty} \frac{6x^5(x^2+4)}{2x(x^2+7)} = 3 \lim_{x \rightarrow \infty} \frac{x^7 + 4x^5}{x^7 + 7x} = 3 \cdot 1 = 3$$

$$5. y = 1 - \ln(x+y) + e^y \text{ 两边同时对 } x \text{ 求导得}$$

$$y' = -\frac{1+y'}{x+y} + y' e^y \Rightarrow y' = -\frac{1}{1+(x+y)(1-e^y)} = \frac{1}{(x+y)(e^y-1)-1}$$

$$6. \lim_{n \rightarrow \infty} \frac{1}{n} (\sin \frac{1}{n} + \sin \frac{2}{n} + \dots + \sin \frac{n}{n}) = \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{n} \sin \frac{k}{n} = \int_0^1 \sin x dx = -\cos x \Big|_0^1 = 1 - \cos 1$$

$$7. \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2}{2t} = \frac{3}{2}t \quad \frac{dy}{dx} \Big|_{t=2} = 3$$

$$\text{当 } t=2 \text{ 时, } x=5, y=8 \text{ 故切线方程为 } y-8 = 3(x-5) \text{ 即 } 3x-y-7=0$$

$$8. y = x \cos x$$

$$\begin{aligned} y^{(5)} &= \sum_{k=0}^5 C_5^k x^{(k)} \cos^{(5-k)} x = C_5^0 x \cos^{(5)} x + C_5^1 x \cos^{(4)} x + 0 \\ &= x \frac{2^5}{2} \cos(2x + \frac{5}{2}x) + 5 \frac{2^4}{2} \cos(2x + \frac{4}{2}x) \\ &= 16x \cdot (-\sin 2x) + 40 \cos 2x \\ &= -16x \sin 2x + 40 \cos 2x \end{aligned}$$

$$9. k = \lim_{x \rightarrow \infty} \frac{y}{x} = \lim_{x \rightarrow \infty} \frac{x^2}{1+x^2} + \frac{\arctan(1+x^2)}{x} \xrightarrow{\text{有界量}} 1 + 0 = 1$$

$$b = \lim_{x \rightarrow \infty} (y - kx) = \lim_{x \rightarrow \infty} (\frac{x^3}{1+x^2} - x + \arctan(1+x^2)) = \lim_{x \rightarrow \infty} \frac{x^3 - x - x^3}{1+x^2} + \frac{\pi}{2} = 0 + \frac{\pi}{2} = \frac{\pi}{2}$$

令该渐近线方程为 $y = x + \frac{\pi}{2}$

二、选择题

1. D

解: 导数定义判断

对于 (D) $f(x) = \cos \sqrt{x}$

$$f'_+(0) = \lim_{x \rightarrow 0^+} \frac{f(x) - f(0)}{x - 0} = \frac{\cos \sqrt{x} - 1}{x} = \frac{-\frac{1}{2}(\sqrt{x})^2}{x} = -\frac{1}{2}$$

$$f'_-(0) = \lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x - 0} = \frac{\cos \sqrt{-x} - 1}{x} = \frac{-\frac{1}{2}(\sqrt{-x})^2}{x} = \frac{1}{2}$$

$$f'_+(0) \neq f'_-(0)$$

所以 $f(x) = \cos \sqrt{x}$ 在 $x=0$ 处不可导

2. C

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{1 - 2e^{\frac{1}{x}}}{1 + e^{\frac{1}{x}}} \arctan \frac{1}{2x} = -2 \cdot \frac{\pi}{2} = -\pi$$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{1 - 2e^{\frac{1}{x}}}{1 + e^{\frac{1}{x}}} \arctan \frac{1}{2x} = 1 \cdot (-\frac{\pi}{2}) = -\frac{\pi}{2}$$

在 $x=0$ 左右极限存在但不相等, 故为跳跃间断点

三、计算题

1. $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{\frac{1}{\sin x}}$

$$= \lim_{x \rightarrow 0} \left(1 + \frac{\sin x - x}{x} \right)^{\frac{x}{\sin x - x}} \cdot \frac{\sin x - x}{x \sin x} = e^{\lim_{x \rightarrow 0} \frac{\sin x - x}{x^2}} = e^{\lim_{x \rightarrow 0} \frac{-\frac{1}{6}x^3 + o(x^3)}{x^2}} = e^{-\frac{1}{6}}$$

2. $\frac{dx}{dt} = \frac{\cos t}{\sin t}$, $\frac{dy}{dt} = -\sin t + \sin t + t \cos t = t \cos t$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{t \cos t}{\cos t / \sin t} = t \sin t$$

$$\frac{d^2y}{dx^2} = \frac{d(\frac{dy}{dx})}{dx} = \frac{d(\frac{dy}{dx})/dt}{dx/dt} = \frac{1 \cdot \sin t + t \cos t}{\cos t / \sin t} = \frac{\sin^2 t + t \sin t \cos t}{\cos t} = \sin t \tan t + t \sin t$$

$$\left. \frac{d^2y}{dx^2} \right|_{t=\frac{\pi}{4}} = \sin \frac{\pi}{4} \tan \frac{\pi}{4} + \frac{\pi}{4} \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} + \frac{\pi}{4} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} \left(1 + \frac{\pi}{4} \right)$$

3. 令 $\sqrt{x} = u$ 则 $u^2 = x$, $dx = 2u du$

$$\int_0^{\frac{\pi^2}{4}} (\sin \sqrt{x})^2 dx = \int_0^{\frac{\pi}{2}} 2u \sin^2 u du = \int_0^{\frac{\pi}{2}} u (1 - \cos 2u) du = \int_0^{\frac{\pi}{2}} u du - \int_0^{\frac{\pi}{2}} u \cos 2u du$$

$$\int_0^{\frac{\pi}{2}} u \cos 2u du = \frac{1}{2} \int_0^{\frac{\pi}{2}} u d \sin 2u = \frac{1}{2} \left(u \sin 2u \Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin 2u du \right) = \frac{1}{2} \left(0 + \frac{\cos 2u}{2} \Big|_0^{\frac{\pi}{2}} \right) = \frac{1}{2} \left(-\frac{1}{2} - \frac{1}{2} \right) = -\frac{1}{2}$$

$$\int_0^{\frac{\pi^2}{4}} (\sin \sqrt{x})^2 dx = \frac{1}{2} \left(\frac{\pi^2}{2} \right) - \left(-\frac{1}{2} \right) = \frac{\pi^2}{8} + \frac{1}{2}$$

$$\begin{aligned}
 4. \lim_{x \rightarrow 0} \left(\frac{\arcsin x}{x} \right)^{\frac{1}{\tan x^2}} \\
 &= \lim_{x \rightarrow 0} \left(1 + \frac{\arcsin x - x}{x} \right)^{\frac{x}{\arcsin x - x} \cdot \frac{\arcsin x - x}{x \tan x^2}} \\
 &= e^{\lim_{x \rightarrow 0} \frac{\arcsin x - x}{x^3}} = e^{\lim_{x \rightarrow 0} \frac{\frac{1}{6}x^3 + o(x^3)}{x^3}} = e^{\frac{1}{6}}
 \end{aligned}$$

$$\begin{aligned}
 5. \int \frac{x e^x}{\sqrt{x+1}} dx &= 2 \int x d\sqrt{x+1} = 2(x\sqrt{x+1} - \int \sqrt{x+1} dx) \\
 \int \sqrt{x+1} dx &\stackrel{\text{令 } t=\sqrt{x+1}}{=} \int t \cdot \frac{2t}{t^2-1} dt = 2 \int (1 + \frac{1}{t^2-1}) dt = 2t + \int \frac{1}{t-1} - \frac{1}{t+1} dt \\
 &= 2t + \ln \left| \frac{t-1}{t+1} \right| + C \\
 \text{故 } \int \frac{x e^x}{\sqrt{x+1}} dx &= 2(x\sqrt{x+1} - 2\sqrt{x+1} + \ln \frac{\sqrt{x+1}+1}{\sqrt{x+1}-1}) + C = 2\sqrt{x+1} + \ln \left| \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1} \right| \\
 &= 2(x-2)\sqrt{x+1} + 4 \ln(\sqrt{x+1}+1) - 2x + C
 \end{aligned}$$

$$\begin{aligned}
 6. \frac{dx}{dt} &= a(\cos t - t \sin t) \quad \frac{dy}{dt} = a(\sin t + t \cos t) \quad \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\sin t + t \cos t}{\cos t - t \sin t} \\
 \frac{dy}{dx^2} &= \frac{d(\frac{dy}{dx})}{dx} = \frac{d(\frac{dy}{dx})/dt}{dx/dt} = \frac{(2\cos t - t \sin t)(\cos t - t \sin t) - (\sin t + t \cos t)(-\sin t - t \cos t)}{(\cos t - t \sin t)^2 \cdot a(\cos t - t \sin t)} \\
 &= \frac{2+t^2}{a(\cos t - t \sin t)^3}
 \end{aligned}$$

$$\begin{aligned}
 7. \lim_{x \rightarrow 0^+} (\cos x)^{\frac{\pi}{x}} &= \lim_{x \rightarrow 0^+} (1 + \cos x - 1)^{\frac{1}{\cos x - 1} \cdot \frac{\pi(\cos x - 1)}{x}} = e^{\lim_{x \rightarrow 0^+} \frac{\pi(\cos x - 1)}{x}} = e^{\lim_{x \rightarrow 0^+} \frac{\pi(-\frac{1}{2}x^2)}{x}} = e^{-\frac{\pi}{2}}
 \end{aligned}$$

$$\begin{aligned}
 8. \int \frac{x^3}{\sqrt{9-x^2}} dx &= \frac{1}{2} \int \frac{-x^2}{\sqrt{9-x^2}} d(-x^2+9) \stackrel{\text{令 } t=9-x^2}{=} \frac{1}{2} \int \frac{t-9}{\sqrt{t}} dt = \frac{1}{2} \int \sqrt{t} - \frac{9}{\sqrt{t}} dt \\
 &= \frac{1}{3} t^{\frac{3}{2}} - 9\sqrt{t} + C \\
 &= \frac{1}{3} (9-x^2)^{\frac{3}{2}} - 9\sqrt{9-x^2} + C
 \end{aligned}$$

四. 解: 由题设可知 $F'(x) = f(x)$

$$\text{且 } F(x)f(x) = F(x)F'(x) = \cos 2x$$

$$\text{故 } \int F(x)F'(x) dx = \int \cos 2x dx$$

$$\text{即 } F(x) = \sin 2x + C, \text{ 由 } F(0)=1 \text{ 得 } C=1, F(x) = \sqrt{\sin 2x+1} = |\sin x + \cos x|$$

$$f(x) = F'(x) = (\sqrt{\sin 2x+1})' = \frac{\cos 2x}{\sqrt{\sin 2x+1}} \quad |f(x)| = \frac{|\cos 2x|}{|\sin x + \cos x|} = \frac{|\cos^2 x - \sin^2 x|}{|\cos x + \sin x|} = |\cos x - \sin x|$$

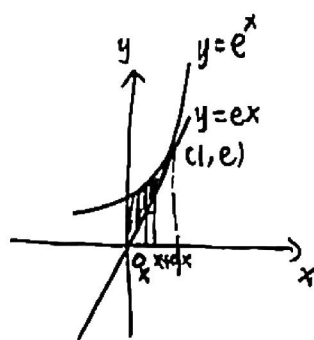
$$\begin{aligned}
 \int_0^{\frac{\pi}{2}} |f(x)| dx &= \int_0^{\frac{\pi}{2}} |\cos x - \sin x| dx = \int_0^{\frac{\pi}{4}} \cos x - \sin x dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x - \cos x dx \\
 &= \sin x + \cos x \Big|_0^{\frac{\pi}{4}} + (-\cos x - \sin x) \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} \\
 &= 2\sqrt{2} - 2
 \end{aligned}$$

五. 解: (1) $y = e^x$ 在 $(1, e)$ 处的切线为 $y - e = e(x - 1)$ 即 $y = ex$

$$S_0 = \int_0^1 (e^x - ex) dx = e^x - \frac{1}{2}ex^2 \Big|_0^1 = \frac{e}{2} - 1$$

(2) 壳层法

$$\begin{aligned} V_0 &= 2\pi \int_0^1 x e^x dx - 2\pi \int_0^1 x \cdot ex dx \\ &= 2\pi \left[(x-1)e^x \Big|_0^1 - \frac{1}{2}ex^3 \Big|_0^1 \right] \\ &= 2\pi \left(1 - \frac{e}{3} \right) \\ &= 2\pi - \frac{2\pi e}{3} \end{aligned}$$



六. 证: 令 $F(x) = f(x) - mx$

因为 $f(x)$ 在 $[a, b]$ 上可导, 则 $F(x)$ 在 $[a, b]$ 上可导且连续, 由闭区间上连续函数的性质, $F(x)$ 在 $[a, b]$ 上可取到最小值

$$F(a) = f(a) - m < 0, \quad F(b) = f(b) - m > 0$$

$F(a) = \lim_{x \rightarrow a^+} \frac{F(x) - F(a)}{x - a} < 0$, 由极限的保号性可知, 存在 $\delta_1 > 0$, 当 $x \in (a, a + \delta_1)$ 时,

$$\frac{F(x) - F(a)}{x - a} < 0, \text{ 即 } F(x) < F(a)$$

$F(b) = \lim_{x \rightarrow b^-} \frac{F(x) - F(b)}{x - b} > 0$, 由极限的保号性可知, 存在 $\delta_2 > 0$, 当 $x \in (b - \delta_2, b)$ 时,

$$\frac{F(x) - F(b)}{x - b} > 0, \text{ 即 } F(x) < F(b)$$

故 $F(x)$ 不在端点 a 和 b 处取得最小值, 而是在 (a, b) 内某点 ξ 处取得.

即存在 $\xi \in (a, b)$, 使得 $\min_{x \in [a, b]} F(x) = F(\xi)$

又根据费马引理, 则有 $F'(\xi) = 0$ 即 $f'(\xi) - m = 0$ 即 $f'(\xi) = m$

$$\begin{aligned} \text{七. 解: } \int_0^{+\infty} \frac{\sin^2 x}{x^2} dx &= - \int_0^{+\infty} \sin^2 x d\left(\frac{1}{x}\right) - \\ &= - \frac{\sin^2 x}{x} \Big|_0^{+\infty} + \int_0^{+\infty} \frac{2 \sin x \cos x}{x} dx \end{aligned}$$

$$= 0 + \int_0^{+\infty} \frac{\sin 2x}{2x} d(2x)$$

$$\stackrel{\text{令 } t=2x}{=} \int_0^{+\infty} \frac{\sin t}{t} dt = \frac{\pi}{2}$$

八、证明:

(1) 证: 令 $g(x) = f(x) + 2023x - 2022$, 则 $g(x)$ 在 $[0, 1]$ 上连续, $(0, 1)$ 内可导.

$$g(0) = f(0) + 0 - 2022 = -2022$$

$$g(1) = f(1) + 2023 - 2022 = 2$$

故 $g(0) \cdot g(1) < 0$, 由零点存在定理, 在 $(0, 1)$ 内至少存在一点 x_0 , 使得 $g(x_0) = 0$

$$\text{即 } f(x_0) = 2022 - 2023x_0.$$

(2) 证: 由 (1) 知存在 $x_0 \in (0, 1)$, 使得 $f(x_0) = 2022 - 2023x_0$.

在 $[0, x_0]$ 和 $[x_0, 1]$ 上对 $f(x)$ 分别应用拉格朗日中值定理

则存在两个不同的点 ξ, η , 且 $\xi \in (0, x_0)$, $\eta \in (x_0, 1)$, 使得

$$f'(\xi) = \frac{f(x_0) - f(0)}{x_0 - 0} = \frac{2022 - 2023x_0}{x_0}$$

$$f'(\eta) = \frac{f(1) - f(x_0)}{1 - x_0} = \frac{2023x_0 - 2021}{1 - x_0}$$

$$\text{故 } [1 + f'(\xi)] \cdot [2021 + f'(\eta)]$$

$$= \left(1 + \frac{2022 - 2023x_0}{x_0}\right) \left(2021 + \frac{2023x_0 - 2021}{1 - x_0}\right)$$

$$= \frac{2022 - 2022x_0}{x_0} \cdot \frac{2023x_0 - 2021x_0}{1 - x_0}$$

$$= \frac{2022(1 - x_0)}{x_0} \cdot \frac{2x_0}{1 - x_0}$$

$$= 4044$$

所以在 $(0, 1)$ 内存在不同的点 ξ, η 使得 $[1 + f'(\xi)] \cdot [2021 + f'(\eta)] = 4044$